

1/EH-29 (i) (Syllabus-2019)

Odd Semester, 2020

(Held in March, 2021)

MATHEMATICS

(Elective/Honours)

(GHS-11)

(Algebra-I and Calculus-I)

Marks : 75

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. (a) Prove that for any three sets A, B, C ,
 $(A - C) \cap (B - C) = (A \cap B) - C$. 4
- (b) Give example of a relation which is—
- (i) reflexive but neither symmetric nor transitive;
 - (ii) reflexive, symmetric but not transitive;
 - (iii) symmetric and transitive but not reflexive;
 - (iv) reflexive and anti-symmetric. 1×4=4

(2)

(c) If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are one-to-one and onto mappings, then prove that $g \circ f: X \rightarrow Z$ is one-to-one and onto and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. 5

(d) If $A = \{x, y, z\}$, then find the power set of A . 2

2. (a) If $f(x) = \frac{1+x}{1-x}$, then prove that $2 \cdot f(x) \cdot f(x^2) = 1 + \{f(x)\}^2$. 4

(b) A survey report reveals that 59% of college students like tea whereas 72% like coffee. Find the possible range of the percentage of college students who like both tea and coffee. 4

(c) Draw the graph of the function

$$f(x) = \begin{cases} 3x + 2, & x < 0 \\ x + 1, & x \geq 0 \end{cases}$$

in the interval $[-2, 2]$. Is this function continuous at $x = 0$? 2+2=4

(d) Use ϵ - δ definition to prove that

$$\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0 \quad 3$$

UNIT—II

3. (a) Show that the matrix

$$A = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$$

is nilpotent and find its index. 2+1=3

- (b) Determine if the following system of equations is consistent and if so, find the solution : 4+3=7

$$2x - y + 3z = 8$$

$$-x + 2y + z = 4$$

$$3x + y - 4z = 0$$

- (c) Reduce the following matrix to normal form and find its rank : 5

$$A = \begin{bmatrix} 0 & 1 & -3 \\ 2 & 1 & 4 \\ 1 & 2 & 1 \end{bmatrix}$$

4. (a) Obtain the inverse of the matrix

$$\begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 1 \\ 0 & 5 & -2 \end{bmatrix}$$

by using elementary operations. 6

- (b) Show that every square matrix A can be uniquely expressed as $P + iQ$ where P and Q are Hermitian matrices. 4

- (c) If A and B be two matrices conformable to form the product AB , then show that $(AB)^T = B^T A^T$, where X^T represents the transpose of the matrix X . 5

UNIT—III

5. (a) Find $\frac{dy}{dx}$ (any two) : 4×2=8

(i) $(\cos x)^y = (\sin y)^x$

(ii) $x^3 + y^3 = 3axy$

(iii) $y = (\sec x)^{\tan x}$

- (b) Find the derivative of $y = e^{\sqrt{x}}$ from first principle. 3

- (c) Find the slope of the curve given by $x^2 + y^2 + 2x - 4y - 20 = 0$ at $(2, 6)$. 2

- (d) Evaluate the derivative of x^7 with respect to x^4 . 2

6. (a) Let $y = \tan^{-1} x$. Show that—

(i) $(1 + x^2)y' = 1$

(ii) $(1 + x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$
2+4=6

- (b) Using L' Hospital's rule, evaluate the following (any two) : 3×2=6

(i) $\lim_{x \rightarrow 0} (\sin x)^{2 \tan x}$

(ii) $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$

(iii) $\lim_{x \rightarrow 0} x \log x$

- (c) The radius of a circle is increasing at the rate of 2 cm per second. At what rate is the area increasing when the radius is 10 cm? 3

UNIT—IV

7. (a) Evaluate (any one) : 3

(i) $\int \frac{x^2}{(a + bx)^3} dx$

(ii) $\int \sqrt{\frac{x-1}{x+1}} dx$

- (b) Evaluate (any one) : 4

(i) $\int_0^1 \frac{dx}{(1+x^2)^{3/2}}$

(ii) $\int_0^{\pi/2} \frac{\sin x dx}{\sin x + \cos x}$

- (c) Using the definition of definite integral as the limit of the sum, evaluate

$$\int_0^2 (x^2 + 1) dx \quad 5$$

- (d) Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$, if it converges. 3

8. (a) Show that

$$\int_0^{\pi/2} \log \sin x dx = \frac{\pi}{2} \log \frac{1}{2} \quad 4$$

(b) Evaluate : 5

$$\lim_{n \rightarrow \infty} \left\{ \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n} \right\}$$

(c) If

$$I_n = \int_0^{\pi/2} \sin^n x dx,$$

where n is a positive integer, $n > 1$;
then prove that

$$I_n = \frac{n-1}{n} I_{n-2}$$

Hence evaluate $\int_0^{\pi/2} \sin^5 x dx$. 3+3=6

UNIT—V

9. (a) Show that $v = \frac{A}{r} + B$ satisfies the differential equation

$$\frac{d^2v}{dr^2} + \frac{2}{r} \frac{dv}{dr} = 0 \quad 2$$

(b) Solve : 5

$$\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$$

(c) Show that the equation

$$x dx + y dy + \frac{xdy - ydx}{x^2 + y^2} = 0$$

is exact. 2

(d) Solve any two of the following : 3×2=6

(i) $xy^2 dy - y^3 dx + y^2 dy = dx$

(ii) $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$

(iii) $\frac{dy}{dx} = \frac{6x - 2y - 7}{3x - y + 4}$

10. (a) Find the differential equation of the family of curves

$$y = e^x(a \cos x + b \sin x)$$

where a and b are arbitrary constants. 2

(b) Solve completely $y = px + \frac{a}{p}$ where

$$p = \frac{dy}{dx} \quad 4$$

(c) Find the orthogonal trajectories of the series of hyperbolas $xy = a^2$. 3

(d) Solve any two of the following : 3×2=6

(i) $(D^2 - D - 2)y = e^{2x}$

(ii) $(D^2 - 8D + 15)y = 0$

(iii) $(D^3 - 1)y = 0$
